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Matrix elements and parton showers in hadronic interactions

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Abstract: A method is suggested to combine tree level QCD matrix elements for the production of multi jet final states and the parton shower in hadronic interactions. The method follows closely an algorithm developed recently for the case of e^+e^- annihilations [1].

Keywords: QCD, Deep Inelastic Scattering, Jets, Hadronic Colliders.

Contents

1. Introduction: ME vs. PS

The production of multi jet final states is one of the searching grounds for new physics at current and future collider experiments. Therefore their Monte Carlo simulation, both for signal and background processes is of crucial importance for the success of the experiments. Currently, two alternative ways to fill the phase space for particle emission can be formulated:

1. One might use exact matrix elements (ME) at some given perturbative order in the coupling constant(s), say α_s. In most cases, these MEs are available at the tree level, i.e. at the lowest perturbative order for the production of a specific final state. Then, the final state particles are usually identified with jets and their phase space is cut accordingly. In some cases higher perturbative orders are known as well, and blur this simple correspondence. However, in this paper we will focus on tree level MEs, therefore we can stick to the simple identification of partons with jets. The virtue of the MEs is that they are exact and take *all* interference effects at the perturbative order into account. The downside is that more and more diagrams have to be considered with rising numbers of outgoing particles populating a multidimensional phase space. Clearly, our calculational abilities do not live up to this situation. Furthermore, for

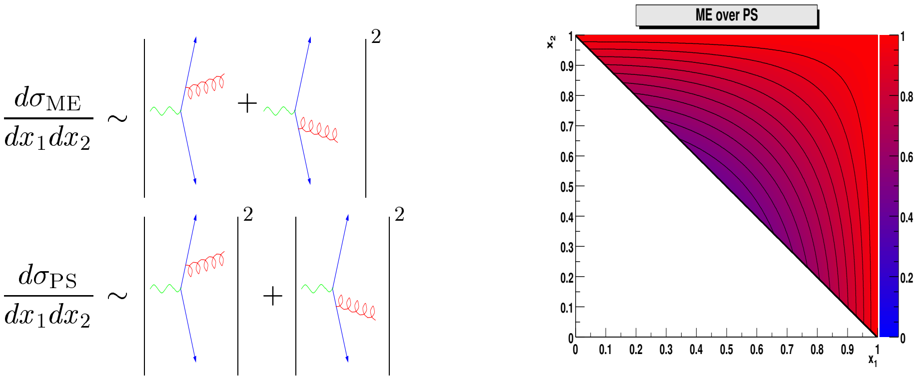

Figure 1: Pictorial representation of contributions to single gluon emission in $e^+e^- \to q\bar{q}g$ as given by the ME and the PS. As indicated, the PS does not take into account the interference contributions present in the ME. This leads to the ratio ME/PS as depicted in the contour plot. At the soft and collinear boundaries of the phase space in the x_1-x_2 plane (where x_i corresponds to the energy fraction of the quark and anti-quark) the PS correctly reproduces the ME, whereas in the region of hard gluon emission the PS omits the (destructive) interference contribution.

the transition of the partons into hadrons a phenomenological fragmentation model has to be invoked with parameters, that are *a priori* free and depend on the fragmentation scale. Hence, applying a fragmentation scheme immediately at the hard scale of the ME necessitates a re-tuning of its parameters for *each* c.m. energy in order to gain reliable results on the hadron level. This is an impossible task and limits the applicability of using pure MEs considerably.

2. Alternatively one might use the parton shower (PS). By expanding around the soft and collinear limits of parton emission the radiation pattern factorises into individual parton branchings, each giving rise to potentially large soft and collinear logarithms. Via multiple emission according to the individual branching probabilities the PS then resums all the large logarithms. In fact, the coherent, angular ordered PS is correct up to Next–to Leading Logarithmic (NLL) accuracy, i.e. it correctly resums *all* terms of the form $\alpha_s^n \left(L^{2n} + L^{2n-1}\right)$, where L denotes a large logarithm of the form $\ln s/Q_0^2$ with s the hard scale of the process and Q_0^2 the soft parton resolution scale. Obviously, the PS is able to connect both the hard scale and the fragmentation scale. Hence, using the PS allows us to tune the fragmentation parameters at one scale, say at LEP1 energies, where huge statistics were amassed, and to use the same parameters at *all* other scales. However, due to its factorised structure the PS misses important interference effects that may play a significant role for detailed analyses.

To gain some insight into the effect of these interference effects, let us compare the emission of a gluon in $e^+e^- \to q\bar{q}$. Pictorially we can write as in figure 1.

From the figure above, it can be seen clearly, that in the ratio $d\sigma_{\rm ME}/d\sigma_{\rm PS}$ the omitted interference terms have a large impact in the region where the gluon is hard and emitted at a large angle. Obviously it would be of great interest to combine both ME and PS to take full advantage of their respective virtues. A number of attempts in this direction were

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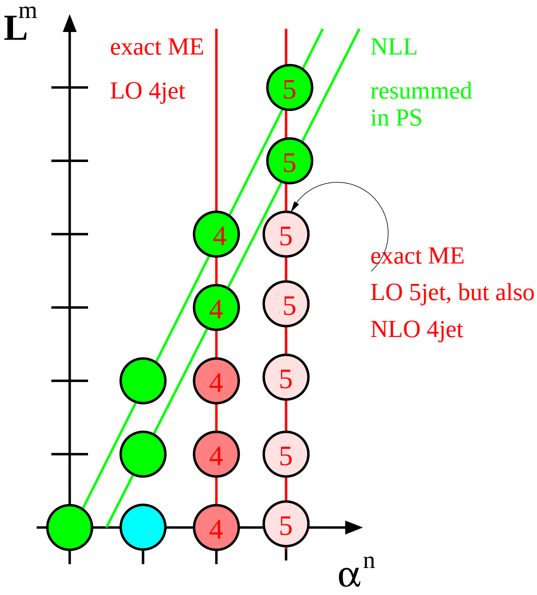

Figure 2: Schematic picture of perturbative orders in $e^+e^- \to$jets. Clearly, for every order in α_s one additional gluon can be emitted that can be soft or collinear leading to maximal two more large logarithms.

made in [2]–[11], some of them also incorporate NLO corrections for specific processes. For the latter, a recent paper [12] provides a general method to construct process specific algorithms to combine MEs at NLO with the PS. This method, however, is limited by the number of processes that are calculated at NLO and that the user wants to implement.

In contrast, within the framework of this paper I would like to restrict the discussion on the merging of MEs at tree level with the PS. This allows for a process independent method that can be applied immediately. In fact, what I propose in this paper is the extension of a previous paper [1], addressing the merging of arbitrary MEs for jet production at tree level and the PS in e^+e^- annihilations, to processes with hadronic initial states.

The basic idea of this method is to divide the phase space for parton radiation in two disjoint regions, one region of jet production that is filled with help of MEs and a complementary region of jet production, that is filled with the PS. The combined ME+PS then should correctly reproduce all large logarithms up to NLL accuracy *and* the sub-leading terms present in the ME. Consider as an example the process $e^+e^- \to$ jets. The respective orders in α_s and L are given in figure 2. For the case of four jet production, the method incorporates all the contributions labelled with a 4, i.e. the full tree level ME, *plus all* NLL contributions of higher perturbative order, represented by the green blobs to the right of the vertical four jet line.

The challenge in such a procedure is to avoid double counting. In our example of four jet production this translates into ensuring that the green blobs labelled by a 4 are taken into account only once. This can be achieved by applying weights on the tree level ME such that the NLL terms of higher perturbative order in α_s are correctly treated and by a veto on the production of extra jets in the PS. To see how this works, let us stick first to the example of jet production in e^+e^- annihilations before going to more general cases.

2. ME+PS in e^+e^- annihilations

2.1 Jet rates at NLL accuracy

I'd like to begin the discussion of the combination procedure with a brief review of the PS. There, individual branchings, i.e. parton emissions, are governed by the Sudakov form factor,

$$\Delta(T,t) = \exp\left\{ -\int_t^T \frac{dt'}{t'} \int_{z_-(t')}^{z_+(t')} dz \frac{\alpha[p_\perp(z,t')]}{2\pi} P_{a\to bc}(z) \right\}, \tag{2.1}$$

yielding a *probability for no branching* of parton a into partons b and c between scales T and t. In eq. (2.1) the scales T and t might denote virtual masses like for instance in PYTHIA [13] or scaled opening angles like in HERWIG, [14]. z denotes the energy fraction of the parton after branching, the limits on the z-integration depend on the meaning of the scale parameters as does the particular form of the transverse momentum $p_\perp$ in the argument of α_s, and the splitting function $P_{a\to bc}$ depends on the parton types involved. In the following we will consider only the angular ordered, coherent PS [15]. Then the scales T and t play the role of scaled opening angles, i.e. the branching scale of parton a, t_a reads

$$t_a = E_a^2(1-\cos\theta_{bc}) \tag{2.2}$$

and the z-limits are given by

$$\sqrt{\frac{t_0}{t_a}} < z < 1 - \sqrt{\frac{t_0}{t_a}} \tag{2.3}$$

where t_0 is connected to a minimal branching angle.

However, up to NLL accuracy we can then perform the z-integrals in eq. (2.1) yielding the NLL-Sudakov form factor

$$\Delta_{q,g}^{\rm NLL}(Q,q) = \exp\left[-\int_q^Q dq' \Gamma_{q,g}(q',Q) \right] \tag{2.4}$$

with the integrated splitting functions

$$\begin{aligned} \Gamma_q(q,Q) &= \frac{2C_F}{\pi}\frac{\alpha_s(q)}{q}\left(\log\frac{Q}{q} - \frac{3}{4}\right), \\ \Gamma_g(q,Q) &= \frac{2C_A}{\pi}\frac{\alpha_s(q)}{q}\left(\log\frac{Q}{q} - \frac{11}{12}\right). \end{aligned} \tag{2.5}$$

In eqs. (2.4) and (2.5) I have explicitly denoted the parton type — quark (q) or gluon (g) — and the scales Q and q are now directly related to transverse momenta. For further details I would like to refer to [16].

Now we are in the position to give expressions for n jet rates in e^+e^- annihilations in the Durham — or $k_\perp$ scheme [17]. In this scheme two partons belong to different jets, if

$$y_{ij} = \frac{2\min\{E_i^2, E_j^2\}(1-\cos\theta_{ij})}{s} > y_{\rm jet}, \tag{2.6}$$

where the parameter $y_{\rm jet}$ regulates the "hardness" of the jets and $s = E_{\rm cm}^2$ is the c.m. energy squared of the e^+e^- pair. Defining the jet resolution scale

$$Q_{\rm jet} = \sqrt{y_{\rm jet}} E_{\rm cm} \tag{2.7}$$

we have

$$\begin{aligned}
\mathcal{R}_2^{NLL} &= \left[\Delta_q^{\rm NLL}(E_{\rm cm}, Q_{\rm jet})\right]^2 \\
\mathcal{R}_3^{NLL} &= 2\left[\Delta_q^{\rm NLL}(E_{\rm cm}, Q_{\rm jet})\right] \times \\
&\quad \times \int_{Q_{\rm jet}}^{E_{\rm cm}} dq \frac{\Delta_q^{\rm NLL}(E_{\rm cm}, Q_{\rm jet})}{\Delta_q^{\rm NLL}(q, Q_{\rm jet})} \Gamma_q(q, E_{\rm cm}) \Delta_q^{\rm NLL}(q, Q_{\rm jet}) \Delta_g^{\rm NLL}(q, Q_{\rm jet}) \\
&= 2\left[\Delta_q^{\rm NLL}(E_{\rm cm}, Q_{\rm jet})\right]^2 \int_{Q_{\rm jet}}^{E_{\rm cm}} dq \Gamma_q(q, E_{\rm cm}) \Delta_g^{\rm NLL}(q, Q_{\rm jet})
\end{aligned} \tag{2.8}$$

The interpretation is quite simple: Since on the parton level jet rates are mere probabilities, the two jet rate is just the combined probability that neither the quark nor the anti-quark have emitted another parton at scales above the jet resolution scale. In turn the three jet rate is a combination of two possible "histories". In either history, one quark does not emit a parton resolvable at $Q_{\rm jet}$, whereas the other one first propagates down to an intermediate scale q without having radiated a parton resolvable at $Q_{\rm jet}$. Then, at q it decays into a quark and a gluon and both decay products experience an evolution down to the jet resolution scale without branching any further. That way, the rates for higher jet configurations can be constructed as well, for details see [17]. However, let us note that omitting the integral over q differential jet rates at NLL can be constructed, i.e. expressions like $d\mathcal{R}_3/dq$.

2.2 Differential jet rates and the ME weight

This leads us directly to the construction of the weights on the ME. Having chosen the final state momenta p_i according to the ME with α_s taken at the jet resolution scale the weight is construct along the following lines:

1. Construct the correct "PS history":

   - Merge the two partons i and j with the smallest y_{ij}. Combine only "allowed" pairs, like for instance {quark, gluon} or {quark, anti-quark}. The momentum of the new parton is the sum of the momenta of i and j.
   - Repeat the step above until only a $q\bar{q}$ pair remains.

2. For each parton line of type p between $q_{\rm in}$ and $q_{\rm out}$ apply a weight

$$\frac{\Delta_p^{\rm NLL}(q_{\rm in}, Q_{\rm jet})}{\Delta_p^{\rm NLL}(q_{\rm out}, Q_{\rm jet})}, \tag{2.9}$$

   where $q_{\rm out}$ might be $Q_{\rm jet}$ for outgoing partons.

3. For each QCD node apply a correction factor

$$\frac{\alpha_s(q_{\rm node})}{\alpha_s(Q_{\rm jet})}\,. \tag{2.10}$$

4. Accept or reject the kinematical configuration according to the combined weight.

As an example let us consider three jet production as in figure 3. In this case the correction weight reads

$$\begin{aligned}\mathcal{W} = &\ \Delta_q(Q_{\rm jet},Q)\frac{\Delta_q(Q_{\rm jet},Q)}{\Delta_q(Q_{\rm jet},q)}\frac{\alpha_s(q)}{\alpha_s(Q_{\rm jet})}\times\\ &\times\Delta_q(Q_{\rm jet},q)\Delta_g(Q_{\rm jet},q)\end{aligned}$$

yielding the correct jet rate $\mathcal{R}_3$ after taking into account configurations where the gluon gets clustered with the other quark line and after inserting the appropriate integrated splitting function and integration over all scales q.

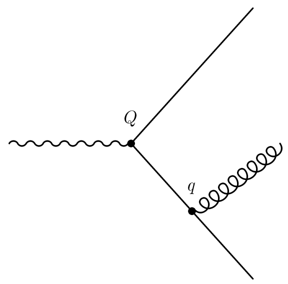

Figure 3.

In this way, all the integrated splitting functions of the differential jet rates at NLL accuracy have been replaced by the correct ME. This leads to an improved description of the region y_{qg}, $y_{\bar{q}g} > y_{\rm jet}$, i.e. the region of hard jet emission where interference effects matter more and more. Clearly the combined weight of this procedure and the ME takes into account the full ME for jet production at tree-level plus all the higher order leading and next-to leading logarithmic contributions.

From the considerations above we can formulate the re-weighting procedure as a cookbook recipe as follows:

1. Expand the PS weight up to the perturbative order in α_s for jet production at tree level. In fact this boils down to a product of integrated splitting functions.

2. Replace this leading term of the result with the full ME squared.

Let me note in passing that this is exactly the approach utilised by PYTHIA [13] and HERWIG [14] but only up to the first order in α_s. In both programmes, either the first or the hardest emission off both the quark and the anti-quark line is re-weighted with the ME,[1] thus suitably replacing the splitting function. The re-weighting procedure makes use of the fact, that $d\sigma_{\rm ME}(q\bar{q}g) < d\sigma_{\rm PS}(q\bar{q}g)$ in the full phase space of gluon emission. This relation is not valid for higher parton configurations any more and consequently the re-weighting procedure has to be modified. The most obvious way to keep the re-weighting applicable is to enhance the value of α_s in the Sudakov form factor, $\alpha_s \to k\cdot\alpha_s$ with $k > 1$ a suitable constant factor. However, due to a potentially large number of rejected emissions, this modification might lead to tremendously inefficient parton showers. For more details on the rejection procedure in various processes see [5]–[7].

[1]The weight is given by $d\sigma_{\rm ME}(q\bar{q}g)/d\sigma_{\rm PS}(q\bar{q}g)$.

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2.3 Vetoing the PS

Having corrected the ME on differential jet rates exact up to NLL accuracy, we are now left with the task of performing the PS. The question then naturally arises, at which scale to start the PS. Naively one might try to start the parton shower at the jet resolution scale, i.e. at $Q_{\rm jet}$. However, this is wrong. A naive counter argument is that in such a case, there would be a radiation dip at scales just below $Q_{\rm jet}$. For a more formal argument, consider two jet events. Their rate is

$$\mathcal{R}_2^{NLL}(Q_{\rm jet}) = \left[\Delta_q^{\rm NLL}(E_{\rm cm}, Q_{\rm jet})\right]^2 , \tag{2.11}$$

see eq. (2.8). The probability for a two jet event at an even smaller resolution scale $Q_0 < Q_{\rm jet}$ should read

$$\mathcal{R}_2^{NLL}(Q_0) = \left[\Delta_q^{\rm NLL}(E_{\rm cm}, Q_0)\right]^2 . \tag{2.12}$$

If we start the PS at the jet resolution scale, however, the probability for two jet events at scale Q_0 would read

$$\left[\Delta_q^{\rm NLL}(E_{\rm cm}, Q_{\rm jet})\Delta_q^{\rm NLL}(Q_{\rm jet}, Q_0)\right]^2 , \tag{2.13}$$

which is clearly wrong. The correct answer is to start the shower at the hard scale $E_{\rm cm}$ and veto all emissions with $q > Q_{\rm jet}$. Identifying vetoed emissions with crosses in figure 4 we then find for one quark line

$$\begin{aligned}
&\Delta_q(E_{\rm cm}, Q_0)\Delta_q(E_{\rm cm}, Q_{\rm jet})\left[1 + \int_{Q_{\rm jet}}^{E_{\rm cm}} dq\, \Gamma_q(q, E_{\rm cm}) + \cdots\right] \\
&\qquad \Longrightarrow \frac{\Delta_q(E_{\rm cm}, Q_0)\Delta_q(E_{\rm cm}, Q_{\rm jet})}{\Delta_q(E_{\rm cm}, Q_{\rm jet})} = \Delta_q(E_{\rm cm}, Q_0)\,,
\end{aligned} \tag{2.14}$$

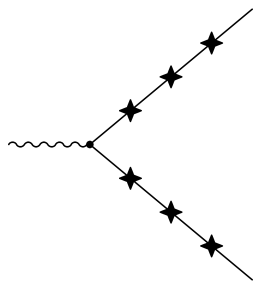

Figure 4.

exactly as demanded. Similar reasoning holds true for more jets, again I want to refer to [1]. This leads to the following algorithm for the PS:

- The PS evolution for *any* outgoing parton starts at the nodal scale $q > Q_{\rm jet}$ where the parton was produced.

- In any branch at some nodal value q, the softer of the two outgoing partons is produced at q whereas the harder parton is produced at some larger scale. This is of specific interest for $g \to gg$ nodes.

Taken everything together it can be shown that in the combination of re-weighting the MEs and vetoing the PS correct NLL jet rates are reproduced at *all* resolution scales above the fragmentation scale. In particular, the dependence on $Q_{\rm jet}$ cancels at NLL accuracy. Furthermore, colour configurations are chosen in a gauge independent way.

3. ME+PS for hadron collisions

For the construction of a combination procedure with — hopefully — similar virtues we employ the same strategy of dividing the phase space for parton emission into a region of jet production and a region of jet evolution. Again, this division is achieved by means of the $k_\perp$-measure [18, 19], and again MEs together with a suitable weight are responsible for the production of jets whereas the PS together with a veto on the emergence of extra jets takes care of the jet evolution down to the fragmentation scale.

3.1 Some brief reminders

Before I go into more detail I'd like to review briefly the PS in the initial state. For definiteness let us constrain ourselves on the case of processes with hadrons as initial states, since the extension to processes involving both hadrons and leptons in the initial state (like for instance Deep-Inelastic Scattering, DIS) is straightforward. First, some process $h_1 h_2 \to X$ and its configuration in phase space is generated according to

$$\sigma_{h_1 h_2 \to X} = \sum_{p_1, p_2} \int dx_1 dx_2 d\hat{\Phi} f_{p_1}^{h_1}(x_1, Q) f_{p_2}^{h_2}(x_2, Q) \frac{d\hat{\sigma}_{p_1 p_2 \to X}}{d\hat{\Phi}}\,, \tag{3.1}$$

where x_i are the energy fractions the partons p_i carry. Furthermore, $d\hat{\sigma}/d\hat{\Phi}$ is the partial differential cross section of the partonic subprocess w.r.t. the differential phase space element $\hat{\Phi}$ of the outgoing particles. The scale Q entering the parton distribution functions f_p^h (PDF) depends on the phase space point under consideration. In Drell-Yan processes, for instance, this scale is given by the invariant mass of the lepton pair, whereas in QCD-type processes this scale is given by the $p_\perp$ of the outgoing partons.

Starting from this scale the PS in the initial scale now proceeds backwards down to some infrared scale Q_0. The backward evolution is due to the fact that in difference to the final state PS here the parton configurations both at the hard and the soft scales are already known. The idea behind the backward evolution is to include the PDFs in order to generate only physically meaningful parton ensembles during the PS evolution.

In this framework, the probability for a parton b to be evolved backwards from (x_2, Q_2) to (x_2, Q_1) with no branching resolvable at the scale Q_0 reads

$$\Pi(Q_1, Q_2; x_2) = \frac{f_b(x_2, Q_1)}{f_b(x_2, Q_2)} \frac{\Delta_b(Q_2, Q_0)}{\Delta_b(Q_1, Q_0)}\,. \tag{3.2}$$

The inclusion of the PDFs actually ensures the correct physical behaviour at low and high values of x_2. Having chosen a scale Q_1, the corresponding value $z = x_2/x_1$ of the splitting $a \to bc$ has to be selected according to

$$\frac{\alpha_s[p_\perp(z, Q_1)]}{2\pi} \frac{x_2/z f_a(x_2/z, Q_1)}{x_2 f_b(x_2, Q_1)} P_{a\to bc}(z)\,. \tag{3.3}$$

After each branching, the system is boosted into the c.m. frame of the two outermost incoming partons.

Let me continue the discussion now with the outline of the algorithm I'd like to propose before I exemplify the construction of the weights applied on the MEs.

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3.2 Proposed algorithm

The algorithm I propose goes as follows:

1. Cluster initial and final state particles backwards according to the longitudinal invariant $k_\perp$ scheme [18, 19] until a $2 \to 2$ process remains. The clustering is achieved step by step in the c.m. system of the incoming partons. In this scheme the initial and final state partons are included in the following fashion:

   - If the two particles considered are both outgoing, their measure y_{ij} is given by

   $$y_{ij} = \frac{2\min\{E_i^2, E_j^2\}(1-\cos\theta_{ij})}{\hat{s}} \to \frac{\min\{p_{\perp,i}^2, p_{\perp,j}^2\}\left[(\eta_i-\eta_j)^2+(\phi_i-\phi_j)^2\right]}{\hat{s}}, \tag{3.4}$$

     see eq. (2.6). Here, $\hat{s}$ is the invariant mass squared of the produced system, $p_\perp$ are the partons' transverse momenta w.r.t. the beam axis and η and ϕ are their pseudorapidities and azimuthal angles.

     If two outgoing, i.e. final state, particles are clustered, the resulting particle again is a final state particle with $p = p_i + p_j$.

   - If one of the two particles, say j is one of the two incoming *partons* then

   $$y_{ij} = \frac{2E_i^2(1-\cos\theta_{ij})}{\hat{s}} \to \frac{p_{\perp,i}^2}{\hat{s}}. \tag{3.5}$$

     If an incoming and an outgoing particle are clustered, the new particle is incoming, and its momentum is $p = p_j - p_i$. Note that in such a case a boost is in order to the c.m. frame of the new pair of two incoming particles.

   In this scheme, the outgoing and incoming particles are clustered "towards" the hard $2 \to 2$ process, i.e. clustering two particles leads to another particle resolved at a higher scale.

   Note that in case we consider DIS-like processes the measures are given by the energies and the cosines, whereas in case of purely hadronic initial states the measures are given in terms of transverse momenta.

2. Find the hardest $k_\perp^2$ in the "core" $2 \to 2$-subprocess. Examples for this hardest $k_\perp^2$ are:

   - $\hat{s} = M_{ll}^2$ in Drell-Yan type $q\bar{q} \to l\bar{l}$ subprocesses.
   - $\frac{2\hat{s}\hat{t}\hat{u}}{\hat{s}^2+\hat{t}^2+\hat{u}^2}$ in QCD subprocesses.

3. Apply a weight constructed by comparing histories.

The rules for the construction of the final weight are:

- For every internal line between two nodal $q_{\rm in} > q_{\rm out}$ values add a factor
$$\frac{\Delta(q_{\rm in}, Q_{\rm jet})}{\Delta_q(q_{\rm out}, Q_{\rm jet})},$$
where again $q_{\rm out}$ might be $Q_{\rm jet}$ for outgonig partons.
- For every QCD node add a factor
$$\frac{\alpha_s(q_{\rm node})}{\alpha_s(Q_{\rm jet})}.$$

Some examples will be given below, special attention will be given on the construction of the PS weights that are to be compared with the ME weights.

4. Start the initial and final state parton showers from the $2 \to 2$-subprocess with corresponding starting conditions, i.e. every leg starts its evolution at the scale where it was produced. This is equivalent to the e^+e^- case already discussed.

5. Veto on jet emissions in the PS. For the vetoes use the same jet algorithm as for the clustering, i.e. for the emission off an outgoing line use the jet measure of eq. (3.4) and for the emissions off an incoming line employ the jet measureof eq. (3.5).

3.3 Constructing the ME weight

In this section I would like to highlight the weight construction, point 3. As an example let us consider Drell-Yan processes with two leptons and two additional jets in the final state.

Similar to the case of e^+e^- annihilations we will first construct a PS weight, that consists of ratios of Sudakov form factors and splitting functions. Let me stress that the graphs in the following examples are not Feynman diagrams but rather pictorial representations of different clustering histories. Note that contributions from the outgoing partons are also included.

3.3.1 "Hard Drell-Yan + 2 softer jets"

Taking $Q_{\rm jet}$ as the jet resolution scale and Q_H as the hardest scale, here $Q_H = M_{ll}$, the PS weight reads for the configuration in the example above

$$\mathcal{W}_{PS} = \frac{q(x_1/z_1, Q_1)}{z_1 q(x_1, Q_H)} \frac{\Delta_q(Q_H, Q_{\rm jet})}{\Delta_q(Q_1, Q_{\rm jet})} \cdot \alpha_s(Q_1) P_{q\to qg}(z_1) \cdot \Delta_g(Q_1, Q_{\rm jet}) \times$$

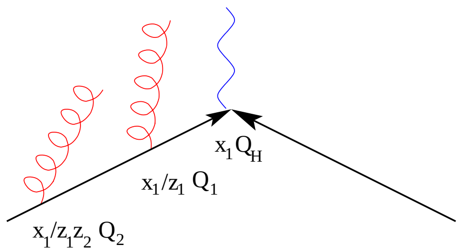

Figure 5: Hard Drell-Yan + 2 gluons.

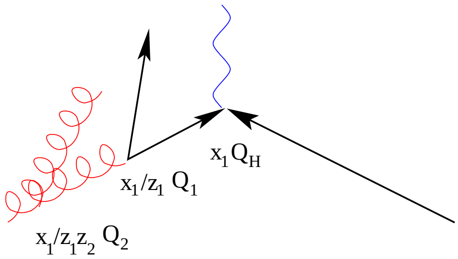

Figure 6: Hard Drell-Yan + quark + gluon.

$$\times \frac{q(x_1/z_1z_2, Q_2)}{z_2 q(x_1/z_1, Q_1)} \frac{\Delta_q(Q_1, Q_{\rm jet})}{\Delta_q(Q_2, Q_{\rm jet})} \cdot \alpha_s(Q_2) P_{q\to qg}(z_2) \cdot \Delta_g(Q_2, Q_{\rm jet}) \times$$
$$\times \frac{q(x_1/z_1z_2, Q_{\rm jet})}{q(x_1/z_1z_2, Q_2)} \frac{\Delta_q(Q_2, Q_{\rm jet})}{\Delta_q(Q_{\rm jet}, Q_{\rm jet})} \times$$
$$\times \frac{q(x_2, Q_{\rm jet})}{q(x_2, Q_H)} \frac{\Delta_q(Q_H, Q_{\rm jet})}{\Delta_q(Q_{\rm jet}, Q_{\rm jet})} \,. \tag{3.6}$$

Note that here and in the following examples I include the Sudakov form factors for the other quark leg and the outgoing partons as well. We have to combine this with the PDFs contained in the expression for the cross section for pure Drell-Yan processes with $Q = Q_H$,

$$d\sigma_{DY} = \int dx_1 dx_2 q(x_1, Q_H) \bar{q}(x_2, Q_H) d\hat{\sigma}_{q\bar{q}\to ll}(\hat{s}, \alpha_s(Q_H)) \,,$$

see eq. (3.1), and compare the result with a similar expression for the production of two additional gluons at a lower scale,

$$d\sigma_{DY+gg} = \int dx'_1 dx_2 q\left(x'_1 = \frac{x_1}{z_1 z_2}, Q_{\rm jet}\right) \bar{q}(x_2, Q_{\rm jet}) d\hat{\sigma}_{q\bar{q}\to llgg}\left(\frac{\hat{s}}{z_1 z_2}, \alpha_s(Q_{\rm jet})\right) .$$

We can easily read off the correction weight on the matrix element, namely

$$\mathcal{W}_{\rm corr.} = \Delta_q^2(Q_H, Q_{\rm jet}) \cdot \frac{\alpha_s(Q_1)}{\alpha_s(Q_{\rm jet})} \cdot \Delta_g(Q_1, Q_{\rm jet}) \cdot \frac{\alpha_s(Q_2)}{\alpha_s(Q_{\rm jet})} \cdot \Delta_g(Q_2, Q_{\rm jet}) \tag{3.7}$$

Note that the missing factor $z_1 z_2$ in the denominator as well as the missing PDF in the numerator is compensated by the differential cross section for the "Drell-Yan + 2 gluons"-process at the jet resolution scale.

Of course, things become slightly more complicated, if some flavour changing branchings like for instance gluon splitting occurs between the jet resolution and the hard scale. For the example above, figure 6, the PS weight would read

$$\mathcal{W}_{PS} = \frac{g(x_1/z_1, Q_1)}{z_1 q(x_1, Q_H)} \frac{\Delta_q(Q_H, Q_{\rm jet})}{\Delta_q(Q_1, Q_{\rm jet})} \cdot \alpha_s(Q_1) P_{g\to qq}(z_1) \cdot \Delta_q(Q_1, Q_{\rm jet}) \times$$
$$\times \frac{g(x_1/z_1z_2, Q_2)}{z_2 g(x_1/z_1, Q_1)} \frac{\Delta_g(Q_1, Q_{\rm jet})}{\Delta_g(Q_2, Q_{\rm jet})} \cdot \alpha_s(Q_2) P_{g\to gg}(z_2) \cdot \Delta_g(Q_2, Q_{\rm jet}) \times$$
$$\times \frac{g(x_1/z_1z_2, Q_{\rm jet})}{g(x_1/z_1z_2, Q_2)} \frac{\Delta_g(Q_2, Q_{\rm jet})}{\Delta_g(Q_{\rm jet}, Q_{\rm jet})} \times$$
$$\times \frac{q(x_2, Q_{\rm jet})}{q(x_2, Q_H)} \frac{\Delta_q(Q_H, Q_{\rm jet})}{\Delta_q(Q_{\rm jet}, Q_{\rm jet})} \,, \tag{3.8}$$

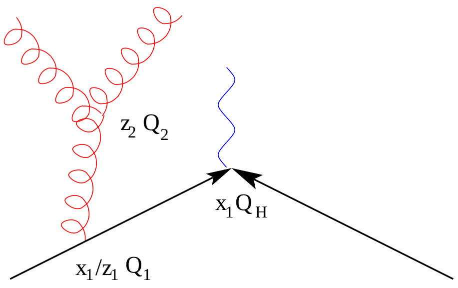

Figure 7: Hard Drell-Yan + splitting gluon.

resulting in the following correction weight

$$\begin{aligned}\mathcal{W}_{\text{corr.}} &= \Delta_q(Q_H, Q_{\text{jet}}) \cdot \Delta_g(Q_1, Q_{\text{jet}}) \cdot \frac{\Delta_q(Q_H, Q_{\text{jet}})}{\Delta_q(Q_1, Q_{\text{jet}})} \cdot \frac{\alpha_s(Q_1)}{\alpha_s(Q_{\text{jet}})} \cdot \Delta_q(Q_1, Q_{\text{jet}}) \cdot \frac{\alpha_s(Q_2)}{\alpha_s(Q_{\text{jet}})} \cdot \\ &\quad \cdot \Delta_g(Q_2, Q_{\text{jet}}) \\ &= \Delta_q^2(Q_H, Q_{\text{jet}}) \cdot \frac{\alpha_s(Q_1)}{\alpha_s(Q_{\text{jet}})} \cdot \Delta_g(Q_1, Q_{\text{jet}}) \cdot \frac{\alpha_s(Q_2)}{\alpha_s(Q_{\text{jet}})} \cdot \Delta_g(Q_2, Q_{\text{jet}})\,. \end{aligned} \tag{3.9}$$

We read off that the correction factor is a combination of Sudakov form factors depending only on the nodes of emissions and the flavours of the external lines. This is in fact quite similar to what we already knew from pure final state considerations.

Let us consider now a case where the "core" $2 \to 2$ subprocess is still of the Drell-Yan type, but where the two softer jets can be clustered to yield a harder one that is still resolved at a scale below the mass of the lepton-pair, see figure 7. In this case, the PS weight reads

$$\begin{aligned}\mathcal{W}_{PS} = & \frac{q(x_1/z_1, Q_1)}{z_1 q(x_1, Q_H)} \frac{\Delta_q(Q_H, Q_{\text{jet}})}{\Delta_q(Q_1, Q_{\text{jet}})} \cdot \alpha_s(Q_1) P_{q\to qg}(z_1) \times \\ & \times \frac{q(x_1/z_1, Q_{\text{jet}})}{q(x_1/z_1, Q_1)} \frac{\Delta_q(Q_1, Q_{\text{jet}})}{\Delta_q(Q_{\text{jet}}, Q_{\text{jet}})} \times \\ & \times \frac{\Delta_g(Q_1, Q_{\text{jet}})}{\Delta_g(Q_2, Q_{\text{jet}})} \cdot \alpha_s(Q_2) P_{g\to gg}(z_2) \cdot \Delta_g^2(Q_2, Q_{\text{jet}}) \times \\ & \times \frac{q(x_2, Q_{\text{jet}})}{q(x_2, Q_H)} \frac{\Delta_q(Q_H, Q_{\text{jet}})}{\Delta_q(Q_{\text{jet}}, Q_{\text{jet}})}\,. \end{aligned} \tag{3.10}$$

It is easy to check that the correction weight is then

$$\mathcal{W}_{\text{corr.}} = \Delta_q^2(Q_H, Q_{\text{jet}}) \cdot \frac{\alpha_s(Q_1)}{\alpha_s(Q_{\text{jet}})} \cdot \Delta_g(Q_1, Q_{\text{jet}}) \cdot \frac{\alpha_s(Q_2)}{\alpha_s(Q_{\text{jet}})} \cdot \Delta_g(Q_2, Q_{\text{jet}})\,. \tag{3.11}$$

3.3.2 "$q\bar{q} \to gV$ + 1 softer jet"

Let us assume now that M_{ll} is not the hardest scale, i.e. the "core" $2 \to 2$ process is not $q\bar{q} \to l\bar{l}$ but rather $q\bar{q} \to g\gamma$. Then the hardest scale in the process is the $p_\perp^2$ of the two bosons and can be approximated by $Q_H^2 \approx (2\hat{s}\hat{t}\hat{u})/(\hat{s}^2 + \hat{t}^2 + \hat{u}^2)$. In this case we are not talking about a PS weight for the process "Drell-Yan + 2 gluons" as in the example above but rather about a "$q\bar{q} \to g\gamma$ + 1 gluon"-process, see figure 8. The corresponding weight reads

$$\begin{aligned}\mathcal{W}_{PS} = {} & \Delta_g(Q_H, Q_{\rm jet}) \\ & \times \frac{q(x_1/z_1, Q_1)}{z_1 q(x_1, Q_H)} \frac{\Delta_q(Q_H, Q_{\rm jet})}{\Delta_q(Q_1, Q_{\rm jet})} \cdot \alpha_s(Q_1) P_{q\to qg}(z_1) \cdot \Delta_g(Q_1, Q_{\rm jet}) \times \\ & \times \frac{q(x_1/z_1, Q_{\rm jet})}{z_2 q(x_1/z_1, Q_1)} \frac{\Delta_q(Q_1, Q_{\rm jet})}{\Delta_q(Q_{\rm jet}, Q_{\rm jet})} \times \\ & \times \frac{q(x_2, Q_{\rm jet})}{q(x_2, Q_H)} \frac{\Delta_q(Q_H, Q_{\rm jet})}{\Delta_q(Q_{\rm jet}, Q_{\rm jet})}\,, \end{aligned} \tag{3.12}$$

with Q_H different from the mass of the lepton pair squared. Then the correction weight reads

$$\mathcal{W}_{\rm corr.} = \Delta_q^2(Q_H, Q_{\rm jet}) \cdot \Delta_g(Q_H, Q_{\rm jet}) \cdot \frac{\alpha_s(Q_1)}{\alpha_s(Q_{\rm jet})} \cdot \Delta_g(Q_1, Q_{\rm jet}) \tag{3.13}$$

3.3.3 "$q\bar{q} \to gg$ + 1 soft lepton pair"

As a last example, let us assume that both gluons are resolved at a scale much higher than the mass of the lepton pair. In this case we have to consider a pure QCD process as the "core" $2 \to 2$ subprocess as exemplified by figure 9. Since neither of the two quark legs experiences any QCD radiation between the hard scale Q_H, again approximated by $Q_H^2 \approx (2\hat{s}\hat{t}\hat{u})/(\hat{s}^2 + \hat{t}^2 + \hat{u}^2)$, the PS weight is simply

$$\begin{aligned}\mathcal{W}_{PS} = {} & \Delta_g^2(Q_H, Q_{\rm jet}) \times \\ & \times \frac{q(x_1, Q_{\rm jet})}{q(x_1, Q_H)} \frac{\Delta_q(Q_H, Q_{\rm jet})}{\Delta_q(Q_{\rm jet}, Q_{\rm jet})} \times \\ & \times \frac{q(x_2, Q_{\rm jet})}{q(x_2, Q_H)} \frac{\Delta_q(Q_H, Q_{\rm jet})}{\Delta_q(Q_{\rm jet}, Q_{\rm jet})}\,, \end{aligned} \tag{3.14}$$

and the correction weight in this case is

$$\mathcal{W}_{\rm corr.} = \Delta_q^2(Q_H, Q_{\rm jet}) \cdot \Delta_g^2(Q_H, Q_{\rm jet}) \tag{3.15}$$

3.4 Vetoed showers

Finally, let us turn to the PS. For the final state particles, the PS evolution is done in the same fashion as in the case of e^+e^- annihilations. For the two initial state lines the

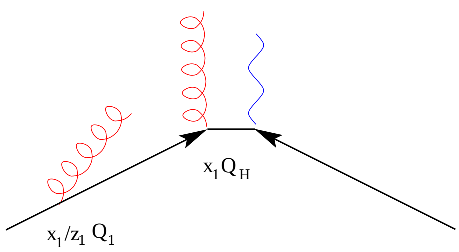

Figure 8: Vector boson production, "$q\bar{q} \to g\gamma^*$".

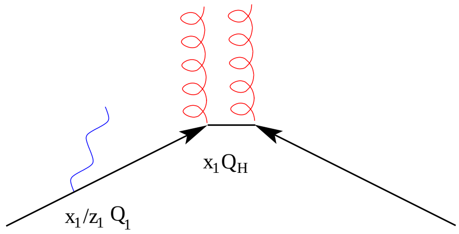

Figure 9: QCD + soft leptons.

evolution from the harder — space-like — scales down to lower scales proceeds as follows:

1. Evolve to lower scales with help of the Sudakov form factors.

2. Veto on all emissions with $q > Q_{\rm jet}$. Here the scale q of the actual emission is defined in the same way as in the clustering procedure, i.e. eq. (3.4) for emissions off an outgoing parton and eq. (3.5) for emissions off an incoming parton.

3. For the first allowed branching with $q_{\rm all.} \le Q_{\rm jet}$ the PDF part of the weight reads:

$$\mathcal{W}_{\rm PDF} = \frac{f(\tilde{x}/\tilde{z}, Q_{\rm all})}{\tilde{z} f(\tilde{x}, Q_{\rm jet})} \,. \tag{3.16}$$

4. Summary and outlook

In this paper I have suggested a method to combine MEs and PS for hadronic interactions. This method is an extension of the already known one for e^+e^- annihilations [1] that has been implemented and successfully tested vs. experimental data in the new event generator AMEGIC++/APACIC++ [20, 21]. In both cases, e^+e^- annihilations and reactions with hadronic initial states, the method correctly takes into account the full perturbative order in α_s of the tree-level ME for the corresponding multi jet production. Suitable starting conditions on the PS and a veto applied on emissions in the PS avoid double counting. In the case of e^+e^- annihilations this method reproduces correctly all contributions up to NLL accuracy by reweighting the ME with Sudakov form factors. In the case of reactions with hadronic initial states such a statement, i.e. the correct resummation of NLL contributions, is far from being trivial and it can be doubted that a complete formal proof like for e^+e^- annihilations into hadrons can be given at all. However, using the method of generating function in the fashion of [18] it might be possible to proof the correctness of the proposed method up to NLL accuracy in $\log Q_H/Q_{\rm jet}$. This is under current consideration.

Finally, I would like to stress that the extension of the method I proposed still waits for the implementation into an event generator. This is also work in progress.

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